Performance Analysis of QPSK and OFDM Systems over AWGN and Frequency-Selective Fading Channels

Ahmed Kamel
Department of Electrical and Computer Engineering
Michigan State University

Abstract—This report presents a comprehensive performance analysis of Quadrature Phase Shift Keying (QPSK) modulation and Orthogonal Frequency Division Multiplexing (OFDM) systems over Additive White Gaussian Noise (AWGN) and frequency-selective fading channels. The study is conducted as part of the MSU ECE 869 Final Project. We investigate the Bit Error Rate (BER) and Symbol Error Rate (SER) through extensive simulations, averaging results over 200 Monte Carlo runs to ensure statistical significance. The impact of AWGN and multipath fading on system performance is analyzed, and theoretical results are compared with simulation outcomes. MATLAB code is developed and referenced throughout the report to illustrate the implementation of the systems under study.

Index Terms—QPSK, OFDM, AWGN, Frequency-Selective Fading, BER, SER, Monte Carlo Simulation, MATLAB.

1 Introduction

Quadrature Phase Shift Keying (QPSK) is a widely used modulation scheme in digital communications due to its bandwidth efficiency and robustness to noise [1]. Orthogonal Frequency Division Multiplexing (OFDM) has emerged as a key technology in modern wireless communication systems, such as LTE and Wi-Fi, due to its ability to mitigate inter-symbol interference (ISI) in frequency-selective fading channels [2].

In this project, we aim to analyze the performance of QPSK and OFDM systems under different channel conditions. Specifically, we address the following:

1) Investigate the BER and SER of a QPSK system over an AWGN channel.
2) Evaluate the performance of an OFDM system employing QPSK modulation under AWGN.
3) Assess the impact of a frequency-selective fading channel on the OFDM system's performance.

Extensive MATLAB simulations are conducted, with code developed to model the transmitter and receiver operations, as well as channel effects. The code is well-commented and modular, facilitating replication and further exploration.

2 Transmitter Design

2.1 QPSK Modulation

The transmitter begins by generating a binary data sequence $\{b_n\}$ of length $L_b = 5120$ bits for each Monte Carlo run. The bits are grouped into pairs and mapped to QPSK symbols using Gray coding to minimize the BER [1]. The symbol mapping is defined as:

$$a_k = \frac{1}{\sqrt{2}} \begin{cases} +1 + j & \text{if } b_{2k} = 0, b_{2k+1} = 0 \\ -1 + j & \text{if } b_{2k} = 0, b_{2k+1} = 1 \\ -1 - j & \text{if } b_{2k} = 1, b_{2k+1} = 1 \\ +1 - j & \text{if } b_{2k} = 1, b_{2k+1} = 0 \end{cases} \tag{1}$$

This mapping ensures that adjacent symbols differ by only one bit, reducing the probability of bit errors.

2.2 OFDM Modulation

For the OFDM system, the QPSK symbols are partitioned into blocks of size $N = 64$, corresponding to the number of subcarriers. Each block $\mathbf{A}_m = [a_0, a_1, \ldots, a_{N-1}]_m$ undergoes an Inverse Fast Fourier Transform (IFFT) to convert the frequency-domain symbols into a time-domain OFDM symbol [3]:

$$\mathbf{S}_m = \text{IFFT}(\mathbf{A}_m) \tag{2}$$

A cyclic prefix of length $L = 5$ is appended to each OFDM symbol to combat ISI caused by multipath fading, ensuring that the linear convolution with the channel becomes circular [2]:

$$\tilde{\mathbf{S}}_m = [s_{N-L}, \ldots, s_{N-1}, s_0, \ldots, s_{N-1}]_m \tag{3}$$

3 Receiver Design

3.1 Reception over AWGN Channel

In the AWGN channel, the received symbols are given by:

$$r_k = a_k + n_k \tag{4}$$

where n_k is a complex Gaussian noise sample with zero mean and variance σ^2 determined by the SNR [1].

3.2 Reception over Frequency-Selective Fading Channel

For the frequency-selective fading channel, the channel impulse response is defined as $\mathbf{h} = [0.3, -0.5, 1, 0, 0.2, -0.3]$, with a channel order $L = 5$. The received signal after passing through the channel is:

$$r_k = \sum_{l=0}^{L} h_l s_{k-l} + n_k \tag{5}$$

At the receiver, the cyclic prefix is removed, and an FFT is performed on each OFDM block to convert the time-domain signal back to the frequency domain:

$$\mathbf{R}_m = \text{FFT}(\mathbf{r}_m) \tag{6}$$

3.3 Channel Equalization

The frequency response of the channel is obtained by performing an N-point FFT on the impulse response $\mathbf{h}$:

$$\mathbf{H} = \text{FFT}(\mathbf{h}, N) \tag{7}$$

Equalization is performed by dividing the received frequency-domain symbols by the channel frequency response:

$$\hat{a}_n = \frac{R_n}{H_n} \tag{8}$$

This zero-forcing equalization compensates for the channel effects but may amplify noise at frequencies where $|H_n|$ is small [2].

3.4 Demodulation and Detection

A minimum-distance detector is employed to map the equalized symbols back to bit pairs. The detected symbol $\hat{a}_k$ is chosen such that:

$$\hat{a}_k = \arg\min_{a \in \mathcal{S}} |\hat{a}_k - a| \tag{9}$$

where $\mathcal{S}$ is the set of QPSK constellation points. The corresponding bits are then assigned based on the Gray coding mapping.

4 Simulation Results

4.1 Simulation Parameters

Simulations were conducted using MATLAB, with the code structured into functions for modularity and clarity. The key parameters are:

- Number of bits per run: $L_b = 5120$
- Number of Monte Carlo runs: 200
- SNR range: 0 to 16 dB in 2 dB steps
- OFDM parameters: $N = 64$ subcarriers, cyclic prefix length $L = 5$
- Channel impulse response: $\mathbf{h} = [0.3, -0.5, 1, 0, 0.2, -0.3]$

The BER and SER are calculated by comparing the transmitted and received bits and symbols, respectively, and averaging over all runs.

4.2 Part I: QPSK over AWGN

The BER and SER for the QPSK system over an AWGN channel were calculated and compared with theoretical values derived from the Q-function [1]:

$$\text{BER}_{\text{theoretical}} = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \tag{10}$$

The simulation results closely match the theoretical predictions, validating the implementation.

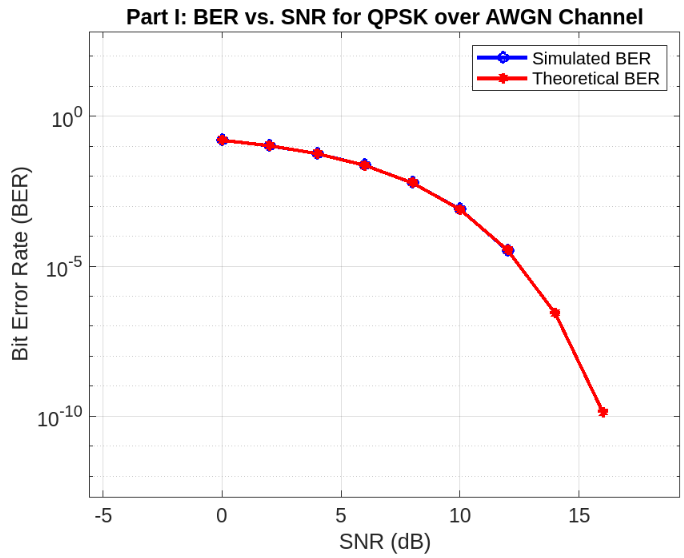

Fig. 1: BER vs. SNR for QPSK over AWGN Channel

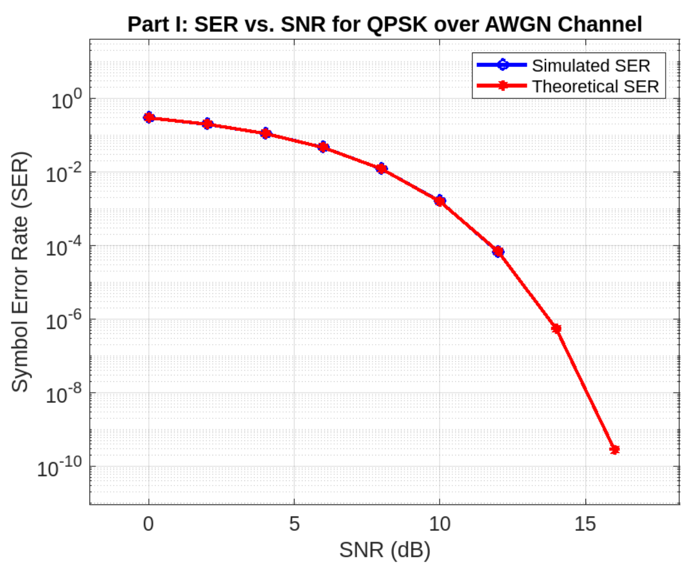

Fig. 2: SER vs. SNR for QPSK over AWGN Channel

4.3 Part II: OFDM under AWGN

The OFDM system's performance over an AWGN channel was evaluated. The results indicate that the BER and SER are consistent with those of the standalone QPSK system, demonstrating that OFDM does not introduce additional errors in AWGN conditions.

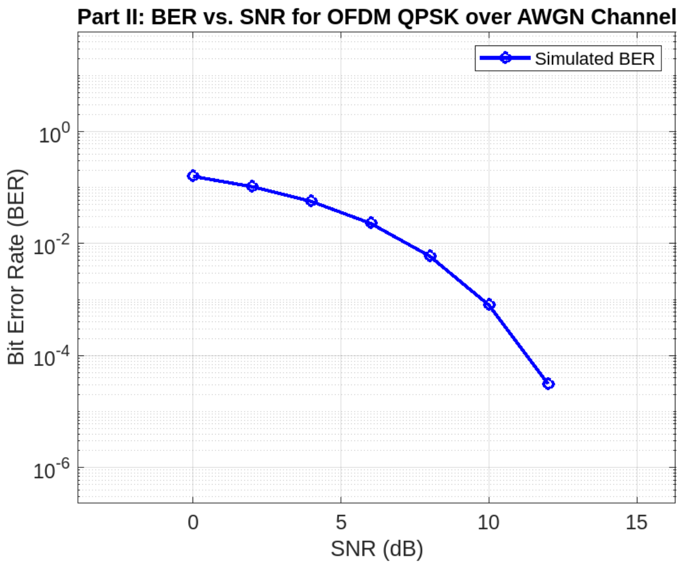

Fig. 3: BER vs. SNR for OFDM QPSK over AWGN Channel

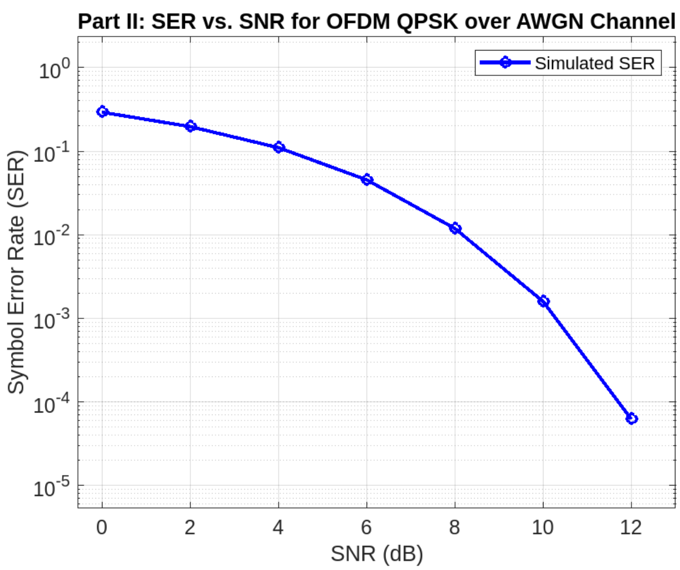

Fig. 4: SER vs. SNR for OFDM QPSK over AWGN Channel

4.4 Part III: OFDM under Frequency-Selective Fading

The impact of the frequency-selective fading channel on the OFDM system's performance was analyzed. The channel introduces ISI, but the use of a cyclic prefix and frequency-domain equalization helps mitigate its effects.

4.5 Comparison of Results

Comparing the performance under AWGN and frequency-selective fading channels reveals that the fading channel degrades the BER and SER due to multipath effects. The zero-forcing equalizer introduces noise enhancement, particularly at frequencies where the channel gain is low [2].

4.6 Discussion

The simulation results confirm that OFDM effectively combats ISI caused by frequency-selective fading when a cyclic prefix is used. However, the performance is still impacted by

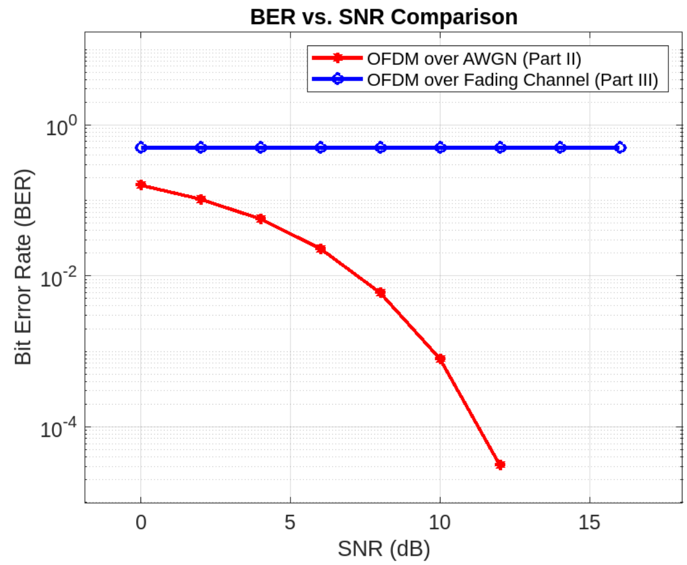

Fig. 5: BER vs. SNR for OFDM QPSK over Frequency-Selective Fading Channel

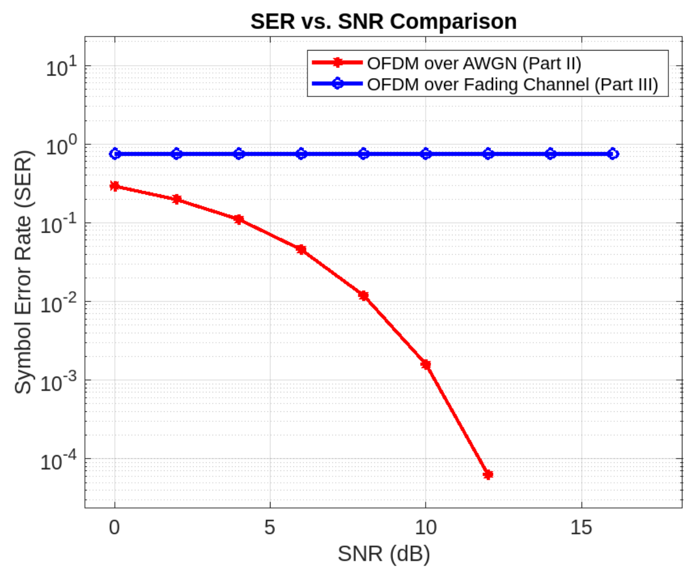

Fig. 6: SER vs. SNR for OFDM QPSK over Frequency-Selective Fading Channel

the channel's frequency response. In practice, more sophisticated equalization techniques, such as Minimum Mean Square Error (MMSE) equalization, can be employed to improve performance [3].

5 Conclusions

This project provided a detailed analysis of QPSK and OFDM systems over AWGN and frequency-selective fading channels. Key conclusions include:

- The QPSK system over AWGN performs as expected, with simulated BER and SER matching theoretical values.
- OFDM over AWGN shows no significant degradation compared to standalone QPSK, validating OFDM's efficacy in additive noise environments.

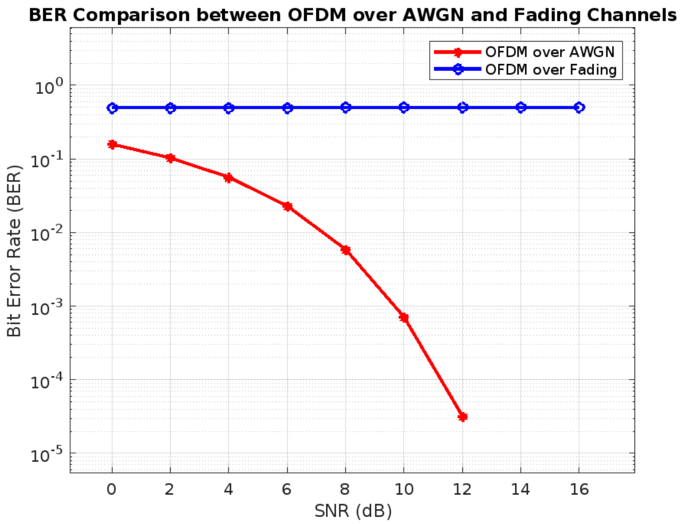

Fig. 7: BER Comparison between OFDM over AWGN and Fading Channels

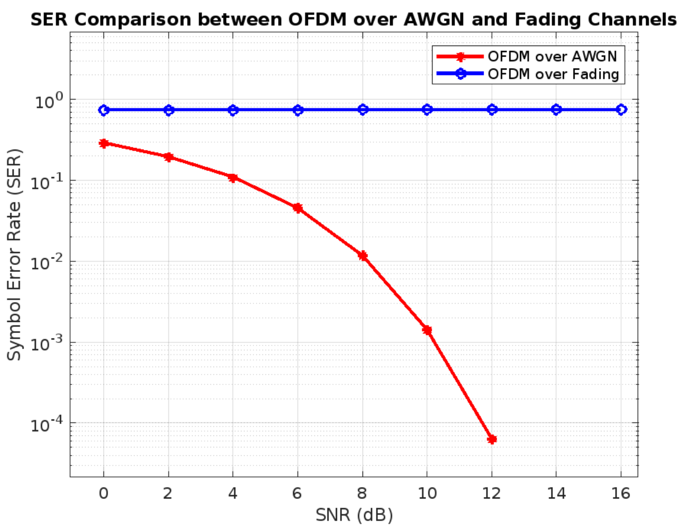

Fig. 8: SER Comparison between OFDM over AWGN and Fading Channels

- Frequency-selective fading introduces challenges due to multipath propagation, but OFDM's use of a cyclic prefix and frequency-domain equalization mitigates ISI.
- Zero-forcing equalization in fading channels results in noise enhancement; alternative equalization methods may offer better performance.

The MATLAB code developed for this project demonstrates the practical implementation of these systems and can serve as a foundation for further exploration, such as incorporating channel estimation errors or using advanced modulation schemes.

References

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